Mark Scheme 4753
June 2005

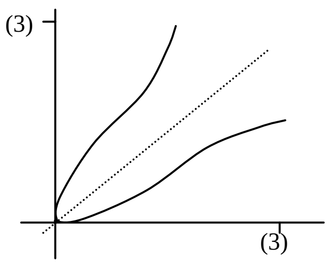
Section A

1 $3x + 2 = 1 \Rightarrow x = –1/3$ $3x + 2 =– 1$ $\Rightarrow x = –1$	B1 M1 A1	$x = –1/3$ from a correct method – must be exact
or $(3x + 2)^2 = 1$ $\Rightarrow 9x^2 + 12x + 3 = 0$ $\Rightarrow 3x^2 + 4x + 1 = 0$ $\Rightarrow (3x + 1)(x + 1) = 0$ $\Rightarrow x = –1/3$ or $x = –1$	M1 B1 A1 [3]	Squaring and expanding correctly $x = –1/3$ $x = –1$
2 $x = ½$ $\cos\theta = ½$ $\Rightarrow \theta = \pi/3$	B1 M1 A1 [3]	 M1A0 for 1.04… or 60°
3 $fg(x) = \ln(x^3)$ $= 3 \ln x$ Stretch s.f. 3 in y direction	M1 A1 B1 [3]	$\ln(x^3)$ $= 3 \ln x$
4 $T = 30 + 20e^0 = 50$ $dT/dt = –0.05 \times 20e^{–0.05t} = –e^{–0.05t}$ When $t = 0$, $dT/dt = –1$ When $T = 40$, $40 = 30 + 20e^{–0.05t}$ $\Rightarrow e^{–0.05t} = ½$ $\Rightarrow –0.05t = \ln ½$ $\Rightarrow t = –20 \ln ½ = 13.86..$ (mins)	B1 M1 A1cao M1 M1 A1cao [6]	50 correct derivative –1 (or 1) substituting $T = 40$ taking lns correctly or trial and improvement – one value above and one below or 13.9 or 13 mins 52 secs or better www condone secs

5 $\int_0^1 \frac{x}{2x+1}dx$ let $u = 2x + 1$ $\Rightarrow du = 2dx,\ x = \frac{u-1}{2}$ When $x = 0$, $u = 1$, when $x = 1$, $u = 3$ $= \int_1^3 \frac{\frac{1}{2}(u-1)}{u}\frac{1}{2}du = \frac{1}{4}\int_1^3 \frac{u-1}{u}du$ $= \frac{1}{4}\int_1^3 (1-\frac{1}{u})du$ $= \frac{1}{4}\left[u - \ln u\right]_1^3$ $= \frac{1}{4}[3 - \ln 3 - 1 + \ln 1]$ $= ¼ (2 – \ln 3)$	M1 A1 B1 M1 A1 E1 [6]	Substituting $\frac{x}{2x+1} = \frac{u-1}{2u}$ o.e. $\frac{1}{4}\int \frac{u-1}{u}du$ o.e. [condone no du] converting limits dividing through by u $\frac{1}{4}\left[u - \ln u\right]$ o.e. – ft their ¼ (only) must be some evidence of substitution
6 $y = \frac{x}{2+3\ln x}$ $\Rightarrow \frac{dy}{dx} = \frac{(2+3\ln x).1 - x.\frac{3}{x}}{(2+3\ln x)^2}$ $= \frac{2+3\ln x - 3}{(2+3\ln x)^2}$ $= \frac{3\ln x - 1}{(2+3\ln x)^2}$ When $\frac{dy}{dx} = 0$, $3\ln x – 1 = 0$ $\Rightarrow \ln x = 1/3$ $\Rightarrow x = e^{1/3}$ $\Rightarrow y = \frac{e^{1/3}}{2+1} = \frac{1}{3}e^{1/3}$	M1 B1 A1 M1 A1cao M1 A1cao [7]	Quotient rule consistent with their derivatives or product rule + chain rule on $(2+3x)^{-1}$ $\frac{d}{dx}(\ln x) = \frac{1}{x}$ soi correct expression their numerator = 0 (or equivalent step from product rule formulation) M0 if denominator = 0 is pursued $x = e^{1/3}$ substituting for their x (correctly) Must be exact: – 0.46… is M1A0
7 $y^2 + y = x^3 + 2x$ $x = 2 \Rightarrow y^2 + y = 12$ $\Rightarrow y^2 + y – 12 = 0$ $\Rightarrow (y – 3)(y + 4) = 0$ $\Rightarrow y = 3$ or $–4$. $2y\frac{dy}{dx} + \frac{dy}{dx} = 3x^2 + 2$ $\Rightarrow \frac{dy}{dx}(2y+1) = 3x^2 + 2$ $\Rightarrow \frac{dy}{dx} = \frac{3x^2+2}{2y+1}$ At (2, 3), $\frac{dy}{dx} = \frac{12+2}{6+1} = 2$ At (2, –4), $\frac{dy}{dx} = \frac{12+2}{-8+1} = -2$	M1 A1 A1 M1 A1cao M1 A1 cao A1 cao [8]	Substituting $x = 2$ $y = 3$ $y = –4$ Implicit differentiation – LHS must be correct substituting $x = 2$, $y = 3$ into their dy/dx, but must require both x and one of their y to be substituted 2 –2

Section B

8 (i) At P, $x \sin 3x = 0$ $\Rightarrow \sin 3x = 0$ $\Rightarrow 3x = \pi$ $\Rightarrow x = \pi/3$	M1 A1 A1cao [3]	$x \sin 3x = 0$ $3x = \pi$ or 180 $x = \pi/3$ or 1.05 or better
(ii) When $x = \pi/6$, $x \sin 3x = \frac{\pi}{6}\sin\frac{\pi}{2} = \frac{\pi}{6}$ $\Rightarrow$ Q($\pi/6$, $\pi/6$) lies on line $y = x$	E1 [1]	$y = \frac{\pi}{6}$ or $x \sin 3x = x \Rightarrow \sin 3x = 1$ etc. Must conclude in radians, and be exact
(iii) $y = x\sin 3x$ $\Rightarrow \frac{dy}{dx} = x.3\cos 3x + \sin 3x$ At Q, $\frac{dy}{dx} = \frac{\pi}{6}.3\cos\frac{\pi}{2} + \sin\frac{\pi}{2} = 1$ = gradient of $y = x$ So line touches curve at this point	B1 M1 A1cao M1 A1ft E1 [6]	d/dx ($\sin 3x$) = $3\cos 3x$ Product rule consistent with their derivs $3x \cos 3x + \sin 3x$ substituting $x = \pi/6$ into their derivative = 1 ft dep 1st M1 = gradient of $y = x$ (www)
(iv) Area under curve = $\int_0^{\frac{\pi}{6}} x\sin 3x\,dx$ Integrating by parts, $u = x$, dv/dx = $\sin 3x$ $\Rightarrow v = -\frac{1}{3}\cos 3x$ $\int_0^{\frac{\pi}{6}} x\sin 3x\,dx = \left[-\frac{1}{3}x\cos 3x\right]_0^{\frac{\pi}{6}} + \int_0^{\frac{\pi}{6}} \frac{1}{3}\cos 3x\,dx$ $= -\frac{1}{3}.\frac{\pi}{6}\cos\frac{\pi}{2} + \frac{1}{3}.0.\cos 0 + \left[\frac{1}{9}\sin 3x\right]_0^{\frac{\pi}{6}}$ $= \frac{1}{9}$ Area under line = $\frac{1}{2}\times\frac{\pi}{6}\times\frac{\pi}{6} = \frac{\pi^2}{72}$ So area required = $\frac{\pi^2}{72} - \frac{1}{9}$ $= \frac{\pi^2 - 8}{72}$ *	 M1 A1cao A1ft M1 A1 B1 E1 [7]	 Parts with $u = x$ dv/dx = $\sin 3x \Rightarrow$ $v = -\frac{1}{3}\cos 3x$ [condone no negative] $\ldots + \left[\frac{1}{9}\sin 3x\right]_0^{\frac{\pi}{6}}$ substituting (correct) limits $\frac{1}{9}$ www $\frac{\pi^2}{72}$ www

9 (i) $f(-x) = \ln[1 + (-x)^2]$ $= \ln[1 + x^2] = f(x)$ Symmetrical about Oy	M1 E1 B1 [3]	If verifies that $f(-x) = f(x)$ using a particular point, allow SCB1 For $f(-x) = \ln(1 + x^2) = f(x)$ allow M1E0 For $f(-x) = \ln(1 +- x^2) = f(x)$ allow M1E0 or 'reflects in Oy', etc
(ii) $y = \ln(1 + x^2)$ let $u = 1 + x^2$ $dy/du = 1/u$, $du/dx = 2x$ $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ $= \frac{1}{u}.2x = \frac{2x}{1+x^2}$ When $x = 2$, $dy/dx = 4/5$.	M1 B1 A1 A1cao [4]	Chain rule $1/u$ soi
(iii) The function is not one to one for this domain	B1 [1]	Or many to one
(iv)  Domain for g(x) $= 0 \le x \le \ln 10$ $y = \ln(1 + x^2)$ $x \leftrightarrow y$ $x = \ln(1 + y^2)$ $\Rightarrow$ $e^x = 1 + y^2$ $\Rightarrow$ $e^x - 1 = y^2$ $\Rightarrow$ $y = \sqrt{(e^x - 1)}$ so $g(x) = \sqrt{(e^x - 1)}$* *or* $g\,f(x) = g[\ln(1 + x^2)]$ $= \sqrt{e^{\ln(1+x^2)} - 1}$ $= (1 + x^2) - 1$ $= x$	M1 A1 B1 M1 M1 E1 M1 M1 E1 [6]	g(x) is f(x) reflected in $y = x$ Reasonable shape and domain, i.e. no –ve x values, inflection shown, does not cross $y = x$ line Condone y instead of x Attempt to invert function Taking exponentials $g(x) = \sqrt{(e^x - 1)}$* www forming g f(x) or f g(x) $e^{\ln(1+x^2)} = 1 + x^2$ or $\ln(1 + e^x - 1) = x$ www
(v) $g'(x) = ½ (e^x - 1)^{-1/2} . e^x$ $\Rightarrow g'(\ln 5) = ½ (e^{\ln 5} - 1)^{-1/2} . e^{\ln 5}$ $= ½ (5 - 1)^{-1/2} . 5$ $= 5/4$ Reciprocal of gradient at P as tangents are reflections in $y = x$.	B1 B1 M1 E1cao B1 [5]	$½\, u^{-1/2}$ soi $\times\, e^x$ substituting ln 5 into g′ - must be some evidence of substitution Must have idea of reciprocal. Not 'inverse'.